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Long-Term Orbit Perturbations of the Draim Four-Satellite Constellations

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Long-term orbit perturbations of the Draim four-satellite constellations were investigated. Results from the 10-yr integration indicate that the orbit variations due to luni-solar perturbations are significant for the two selected orbits with 27- and 48-h periods. The resulting degradation in ground coverage was found to be 16 and 32% for the 27- and 48-h orbit constellations, respectively. Argument of perigee, inclination, and eccentricity controls must be applied to maintain 100% continuous coverage. The maximum ΔV required for a 10-yr mission was estimated to be 835 m/s. A strategy to avoid the costly stationkeeping maneuvers was examined, and the resulting performance improvements were assessed.

Introduction

OVER the past two decades, mission designers have addressed the question, "What is the minimum number of satellites required to insure continuous Earth coverage?" Earlier studies had concluded that this minimum number was six. Later, through mathematical search,[1,2] the minimum number of satellites required to give 100% continuous, one-fold global coverage was found to be five.

More recently, Draim[3,4] has developed geometric theorems and corollaries that led to the discovery of four-satellite constellations that provide 100% continuous global coverage. The orbit period of the four-satellite constellations must be equal to or greater than 26.49 h to insure continuous global coverage.[4] The inclination and eccentricity of the orbits were found to be 31.3 deg and 0.263, respectively. For orbits having mean altitudes higher than those of geosynchronous satellites, the luni-solar gravitational attractions become significant, and the long-term orbit stability should be carefully examined before considering this type of orbit for mission applications.

The inclination and right ascension of the ascending node of a high-altitude orbit are subject to gradual change caused by the sun and moon.[5] The inclination deviation due to luni-solar effects is a function of initial ascending node, and the nodal regression due to oblateness (J_2) effects is a function of the instantaneous value of the inclination. As a result, the perturbation-induced deviations in inclination and node couple with each other, and the accumulated effects on coverage can be significant.[6] Furthermore, the third-body attraction may induce large eccentricity variations for orbits with mean orbit radius larger than that of the geosynchronous orbits.

The purpose of this analysis is to investigate the long-term perturbation effects on the Draim four-satellite constellations with common periods. The required orbit maintenance fuel consumption for offsetting those perturbations will be estimated. A strategy of biasing the initial orbit elements to avoid the costly stationkeeping maneuvers will be examined. The results of the Draim four-satellite constellation performance in the presence of perturbations will be assessed and compared with constellations of four geostationary satellites or Molniya satellites. A Molniya orbit whose period is approximately 12 h has an eccentricity of 0.7, an inclination of 63.4 deg, and an argument of perigee of 270 deg.

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Draim Four-Satellite Constellations

In Ref. 4, Draim derived a four-satellite constellation using common-period elliptic orbits to provide continuous one-fold global coverage. The optimized orbits have a common eccentricity of 0.263 and a common inclination of 31.3 deg. The common orbit period must be equal to or greater than 26.5 h. Two constellations with common periods equal to 27 and 48 h are studied in this analysis. The elliptic orbits are so arranged that two opposing satellites have their perigees in the Northern Hemisphere, whereas the other two have their perigees in the Southern Hemisphere. Figure 1, which is a combination of Figs. 2 and 3 of Ref. 4, shows the orbit geometry of two opposing satellites with two ascending nodes separated by 180 deg. The orbit planes of S_1 and S_3 are parallel to planes ACD and BCD of the tetrahedron, respectively. When S_1 is at its apogee, S_3 is at its perigee, as shown in Fig. 1. Similar geometry exists for the other two satellites, whose orbit planes are parallel to the other two faces of the tetrahedron. The orbit parameters of the two constellations, 27 and 48 h, are listed in Table 1, where a, e, i, Ω, ω, and M are the orbit semimajor axis, eccentricity, inclination, right ascension of ascending node, argument of perigee, and mean anomaly, respectively.

Method of Analysis

To examine the long-term orbit perturbation effects on the Draim four-satellite constellations, orbit histories in terms of four classical elements e, i, Ω, and ω are generated over 10 years using a semianalytic (singly averaged equations of motion) integration program (GEOSYN)[7] with a 4×4 Earth gravity model and sun-moon gravitational attractions. Ground coverage degradations are examined at 1500 and 3000 days after the epoch, using the orbit elements propagated to the two dates.

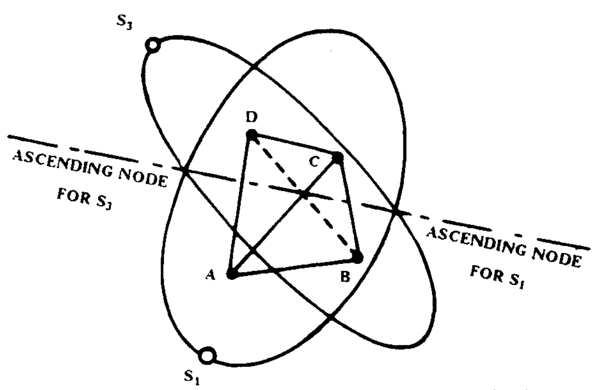

Fig. 1 Draim four-satellite constellation orbit geometry.

Table 1 Orbit parameters of Draim four-satellite constellations

Satellite number	i, deg	e, deg	ω, deg	Ω, deg	M, deg
1	31.3	0.263	270	0	0
2	31.3	0.263	90	90	270
3	31.3	0.263	270	180	180
4	31.3	0.263	90	270	90

a = 45,691.7 km for 27-h orbits; 67,053.6 km for 48-h orbits; a = semimajor axis, e = eccentricity, i = inclination, ω = argument of perigee, Ω = right ascension of ascending node, M = mean anomaly.

Then the integrations of the two constellations (27- and 48-h orbit periods) are repeated using GEOSYN, with stationkeeping ΔV computed by performing simulated inclination and argument of perigee controls. The assumed tolerances for inclination and argument of perigee are ±1 and ±5 deg, respectively. The total ΔV for stationkeeping over 10 years is, to first order, independent of the control tolerances. This is because the ΔV magnitude is linearly proportional to the corrections of the inclination and argument of perigee [see Eqs. (1) and (2)], and the variations of the two angles are an approximately linear function of time (see Figs. 3 and 5).

After studying the long-term orbit histories, coverage degradations, and ΔV consumptions, the initial orbit elements are properly biased to improve the overall coverage performance and minimize the total ΔV requirement. Finally, the coverage results are compared with other four-satellite constellations.

Long-Term Orbit Perturbations

Results of 10-yr integrations of the two selected constellations with a 27- and a 48-h period are shown in Figs. 2–5. Figure 2 shows the histories of eccentricity of the two Draim four-satellite constellations. The four solid curves are the variations of the four orbits with a 27-h period. Similar plots are shown in Figs. 3–5 for inclination, node, and argument of perigee histories, respectively. The common epoch of the 10-yr integration is arbitrarily assumed to be 0 h on November 26, 1995.

It is obvious that the orbit deviations from the initial configuration are significant, especially for the orbits with a 48-h period. The eccentricity can increase to a value as large as 0.42 after 10 years, and the inclination can become as high as 48 deg or as low as 19 deg near the end of the 10-yr mission. The dominant perturbation effects come from the luni-solar attractions and J_2. The histories of the changes from the initial values in right ascension of the ascending node and argument of perigee are plotted in Figs. 4 and 5, respectively. The relative deviations among the four node histories of each constellation imply uneven nodal separations. The deviations are quite large among the orbit planes of the 48-h constellation, as shown by Fig. 4. These uneven separations can be minimized by properly biasing the initial node and inclination values of the four planes. The dispersion in argument of perigee histories is even more pronounced than that of the node as shown in Fig. 5. The large deviations in orbit parameters of the two Draim four-satellite constellations suggest that the degradation in ground coverage can be significant.

Effects on Coverage

It is interesting to see how the changes affect ground coverage. Coverage results were generated on the selected dates at epoch, 1500 days, and 3000 days after epoch with orbit values taken from the output of GEOSYN. The epoch of the numerical integration was arbitrarily chosen as November 26, 1995. The mean anomalies of the two constellations were assumed to have the same values for all cases, as shown in Table 1. It was assumed that in-plane stationkeeping can maintain the same relative phasing throughout the 10-yr simulation period. The coverage results are summarized in Table 2 (no initial biases) and Fig. 6.

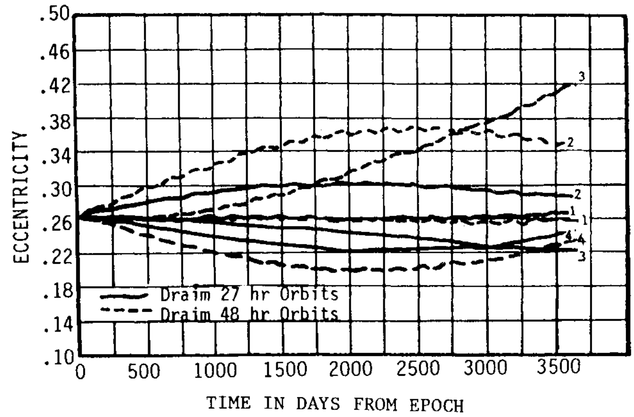

Fig. 2 Eccentricity history of Draim (27 and 48 h) orbits.

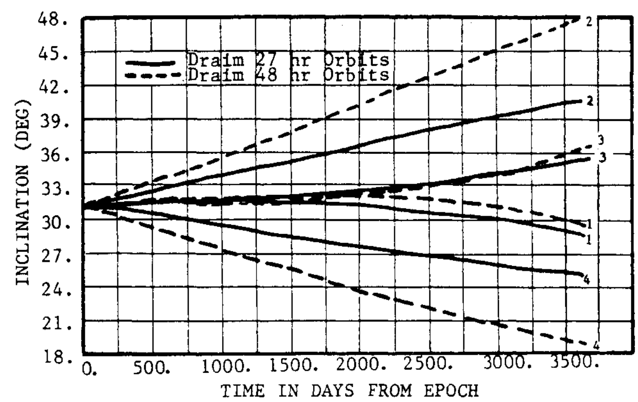

Fig. 3 Inclination history of Draim (27 and 48 h) orbits.

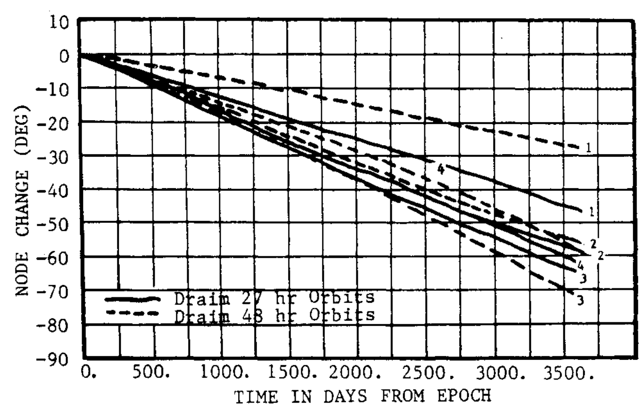

Fig. 4 Change of right ascension of ascending node of Draim (27 and 48 h) orbits.

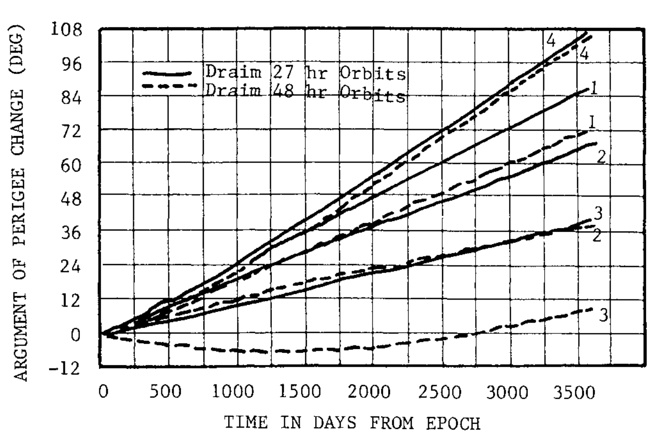

Fig. 5 Argument of perigee change of Draim (27 and 48 h) orbits.

Table 2 Summary of percentage coverage of Draim constellations

		27-h orbit			48-h orbit		
Minimum elevation angle =		0 deg	5 deg	10 deg	0 deg	5 deg	10 deg
Epoch	No initial biases	100	78.4	58.6	100	92.3	46.3
	With initial biases on inclination and node	94.3	78.4	61.9	78.6	64.8	47.8
	With initial biases and control[a]	98.9	79.4	60.2	100	81.5	44.6
1500 days	No initial biases	87.4	75.1	58.7	70.6	48.4	29.7
	With initial biases on inclination and node	95.3	83.5	62.8	98.7	81.9	53.3
	With initial biases and control	99.6	82.6	61.8	99.9	88.4	45.3
3000 days	No initial biases	84.3	78.0	60.2	68.0	53.5	39.7
	With initial biases on inclination and node	91.0	81.0	66.0	78.1	62.7	46.0
	With initial biases and control	97.5	79.9	61.4	90.9	72.9	46.0

[a]Control argument of perigee with a 5-deg tolerance.

For zero elevation angle at epoch, 100% continuous coverage is possible. As the minimum elevation angle increases, the coverage decreases. For the 27-h orbit constellation, the percentage of global continuous coverage with a minimum elevation angle of 5 deg is 78.4%, and the corresponding value with a 10-deg minimum elevation angle is 58.6%. For the 48-h orbit constellation, the percentage of coverage drops to 92.3% for a 5-deg minimum elevation angle and to only 46% when the minimum elevation angle is increased to 10 deg. This sharp drop in continuous ground coverage at 10-deg minimum look angle is because the satellite covers most of the area of one hemisphere during the first 24-h period and most of the area of the other hemisphere during the second 24-h period. Thus, the continuous ground coverage for the 48-h repeater is the limited common region with very low elevation angle that is continuously covered during the 48-h period. The results of the coverage study show that the Draim four-satellite constellations are sensitive to both orbit perturbations and minimum elevation angle. The sensitivity to minimum elevation angle is discussed in Draim's paper,[4] and a plot of minimum elevation angle as a function of orbit altitude is shown in Fig. 5 of Ref. 4.

With the long-term perturbation effects present, the degradations in coverage are significant at 1500 days and 3000 days after epoch for the two Draim constellations, as shown in Fig. 6. For the 27-h constellation, the one-fold coverage with a 0-deg elevation limit has a degradation of 12% 1500 days after epoch and 16% 3000 days after epoch. The corresponding decreases in coverage for the 48-h constellation are 29 and 32%, respectively.

Stationkeeping ΔV Requirements

These results suggest that, in order to maintain 100% continuous coverage, inclination and argument of perigee stationkeeping maneuvers must be applied to the Draim-type constellation. Program GEOSYN simulates the inclination and argument of perigee stationkeeping maneuvers according to specified tolerances. The inclination control is performed at the ascending or descending node with the following equation:

$$\Delta V_i = 2V \sin \frac{\Delta i}{2} \tag{1}$$

where V is the satellite velocity at the node and Δi the required inclination change of each maneuver. The argument of perigee control is performed with the optimal two-burn method[7]:

$$\Delta\omega = \sum_{i=1}^{2} \frac{(2+e\cos f_i)\sin f_i}{ena(1+e\cos f_i)}\left(\frac{\Delta V}{2}\right) \tag{2}$$

where

$$f_i = \cos^{-1}(-e) \pm \frac{e}{2} \quad (i=1 \text{ for the "+"})$$

and where f is the true anomaly, $\Delta\omega$ the change in argument of perigee, and n the orbit mean motion.

The stationkeeping ΔV computed by GEOSYN is based on Eqs. (1) and (2). The required total ΔV and fuel weight for each of the satellites in the two Draim constellations are shown in Table 3. For the Draim-type orbits, the locations of the two optimal burns f_i are close to 90 or 270 deg, and the argument of perigee is either 90 or 270 deg. Therefore, it is possible that the inclination and argument of perigee maneuvers can be combined (vector sum) to save fuel. Results from Table 3 show that the maximum total ΔV required for a 10-yr mission is 835 m/s. For a spacecraft with an initial weight of 3500 lb and a specific impulse of 230 s, the required fuel weight for stationkeeping is as large as 1083 lb if the vector combination is used. Additional fuel may be required to maintain relative phasing and control eccentricity. These maneuvers generally require much less ΔV and may be combined with argument of perigee controls. Therefore, they are not considered in this analysis.

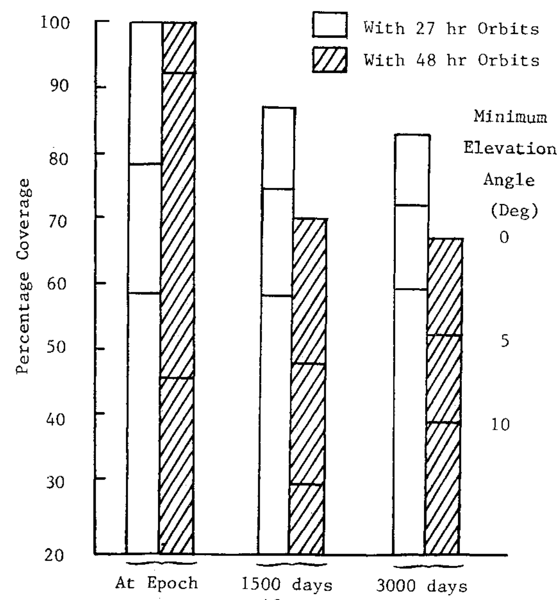

Fig. 6 Draim four-satellite constellation (one-fold) coverage degradations due to orbit perturbations.

Small Biases in the Initial Orbits

One alternative method to minimize coverage loss due to perturbations is to introduce small biases in the initial orbit elements. Those biases can be determined from the long-term histories of the orbit variations. Results of an early analysis[6] show that the global positioning system constellation performance degradation because of perturbations can, in fact, be improved significantly by slightly offsetting the initial inclination and node of each orbit.

Figure 7 gives a comparison of the percentage coverage (one-fold) of the 27-h constellation with and without initial orbit biases. After biasing the initial orbit elements, the coverage degradation has been improved by 7–8%. However, the coverage at epoch drops by 6% due to biasing the initial elements. The improvement is more significant for the 48-h constellation, as shown in Fig. 8. If only the argument of perigee is stationkept with a proper initial biasing of elements, the percentage coverage can be maintained at a very high value, ≥97.5% (see Table 2). The required ΔV for controlling the argument of perigee is as large as 740 m/s (see Table 3).

Comparison with Other Four-Satellite Constellations

These results indicate that the Draim four-satellite constellations are sensitive to long-term orbit perturbations and require a significant amount of fuel to maintain 100% continuous coverage. The results also show that biasing the initial orbit elements can improve the overall coverage to better than 90%; however, 100% continuous global coverage cannot be achieved without the costly stationkeeping maneuvers. From a mission designer's point of view, it is useful to compare the Draim four-satellite constellations with other four-satellite constellations using geosynchronous (circular and near equatorial) or Molniya orbits, whose long-term perturbations are better understood. The dominant long-term inclination perturbations due to sun-moon attractions (0.9 deg/yr) were simulated in the geosynchronous constellation ($0 \leq i \leq 3.5$ deg). The orbit elements assumed for the geosynchronous and Molniya orbits are shown in Table 4.

Figure 9 shows a comparison of percentage of continuous ground coverage among the three four-satellite constellations: Draim 27-h, geosynchronous, and Molniya. Only the in-plane stationkeeping maneuvers with minimal fuel cost are assumed for the latter two constellations to maintain the desired phasing among the satellites. The geosynchronous orbit constellation gives the best overall ground coverage (~95% for zero minimizes elevation) because of its long-term orbit stability. The Draim constellation with a 27-h orbit, initial biases, and argument of perigee control yields results better than that of the geosynchronous constellation when the minimum elevation is zero. However, the Draim constellation and the Molniya constellation are more sensitive to minimum elevation angle, as shown in Fig. 9. A constellation with four Molniya orbits

Table 3 10-yr ΔV and fuel requirements for maintaining Draim constellations

Satellite number		ΔV for inclination control, m/s	ΔV for arg. of perigee control, m/s	Total ΔV (vector sum), m/s	Fuel required, lb
1	27 h	249	620	669	889
	48 h	270	413	494	688
2	27 h	376	740	830	1077
	48 h	590	591	835	1083
3	27 h	231	492	544	750
	48 h	339	242	416	590
4	27 h	262	355	442	623
	48 h	412	126	431	609

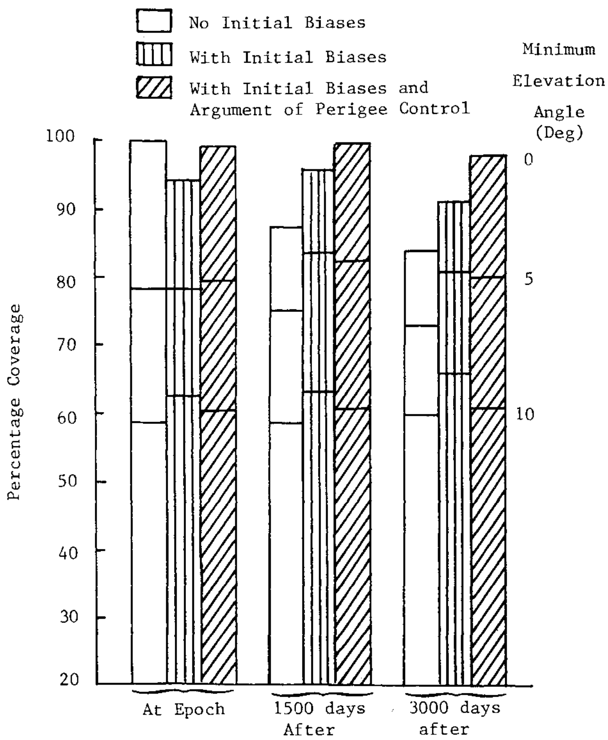

Fig. 7 Draim 27-h constellation one-fold coverage degradations.

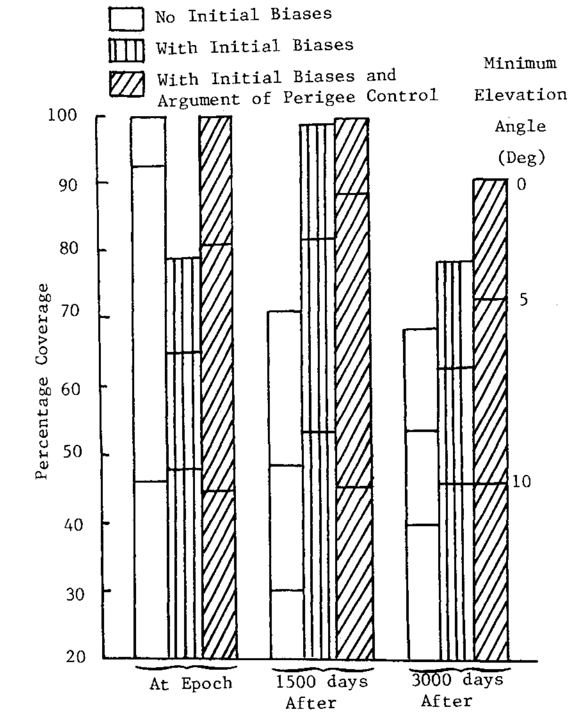

Fig. 8 Draim 48-h constellation one-fold coverage degradations.

Table 4 Orbit parameters of the constellations with geosynchronous and Molniya orbits

Satellite number	Orbit	i, deg	e, deg	ω, deg	Ω, deg	M, deg
1	Geosynchronous	3.5	0.001	0	270	10
	Molniya	63.4	0.7	270	0	0
2	Geosynchronous	0	0.001	0	180	190
	Molniya	63.4	0.7	270	90	180
3	Geosynchronous	1.0	0.001	0	90	10
	Molniya	63.4	0.7	270	180	0
4	Geosynchronous	2.5	0.001	0	0	190
	Molniya	63.4	0.7	270	270	180

$a = 42{,}165.9$ km for geosynchronous orbits; 26,559.5 km for Molniya orbits.

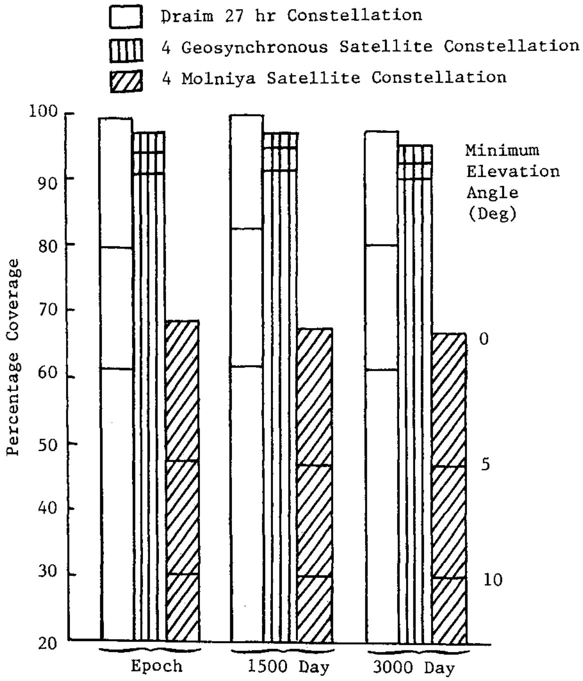

Fig. 9 Comparison of one-fold coverage among three four-satellite constellations.

can only provide 70% continuous coverage of the globe, with the Northern Hemisphere completely covered.

It is important to point out that the initial propellant cost to place a satellite in a geostationary orbit can be significantly more than to inject a satellite into a Draim orbit, due to differences in inclination and eccentricity. For example, an ATLAS II launched from Cape Canaveral can deliver 800 lb of additional payload to a 24-h inclined orbit ($i = 30$ deg) with a 0.2 eccentricity. This additional weight can provide a large part of the stationkeeping fuel needed for maintaining the Draim constellation with 100% coverage.

Other Considerations for Minimizing Perturbation Effects

In this study, the initial biases of the orbit elments of the Draim constellations were determined through iterations, which may not yield the optimal solution for minimizing the perturbation effects. Other approaches for achieving the optimal solution may be to 1) optimize the initial right ascensions of the ascending node as a function of epoch, 2) search for an optimal orientation of the Draim constellation (the tetrahedron) in the inertial space such that the combined effects due to sun/moon and J_2 are minimized over the mission lifetime, and 3) perform periodic in-plane maneuvers to optimize locally the relative spacing to offset perturbations during a short time span. Each of the three approaches requires a considerable amount of effort to analyze and is beyond the scope of this study.

Conclusions

The Draim four-satellite constellations with mean orbit radius greater than geosynchronous distance are subject to significant luni-solar gravitational perturbations. The magnitude of the long-term orbit deviations increases with mean orbit radius. The resulting ground coverage degradations have been found to be 12.5% after 1500 days and 16% after 3000 days for the 27-h constellation. The corresponding degradations in ground coverage for the 48-h constellation are 29.4% after 1500 days and 32% after 3000 days. A different criterion, such as the maximum revisit times, may be used to evaluate the coverage performance. However, it is beyond the scope of this study.

The total ΔV required to control the inclination and argument of perigee for 10 years to maintain 100% continuous coverage is as large as 830 m/s for the 27-h orbit and 835 m/s for the 48-h orbit. Such a high ΔV cost implies a significant increase in payload weight.

Results of this analysis show that the coverage degradation of the 27-h constellation can be largely recovered by properly biasing the initial orbit parameters and only controlling the argument of perigee. This Draim constellation with biased initial orbits and argument of perigee control gives 97.5% or better continuous global coverage, better than a constellation with four geosynchronous satellites. However, when the minimum elevation angle is increased to 5 deg from 0 deg, only 80% coverage is possible, which is less than the 93% coverage by the geosynchronous constellation with the same elevation limit. However, one should note that the geosynchronous constellation does not cover the two polar regions at all, whereas the regions in the Draim constellations that are not continuously covered vary in size, shape, and location with time. The additional fuel needed to place a satellite in the geosynchronous orbit can provide a large part of the stationkeeping propellant required by the Draim constellation. Therefore, further study of the Draim constellation with four satellites is warranted.

Acknowledgments

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References

[1]Walker, J. G., "Continuous Whole Earth Coverage by Circular Orbit Satellite Patterns," Royal Aircraft Establishment, TR 77044, London, UK, March 1977.

[2]Ballard, A. H., "Rosette Constellations of Earth Satellites," *IEEE Transactions on Aerospace and Electronic Systems,* Vol. AES16, No. 5, Sept. 1980, pp. 656–665.

[3]Draim, J. E., "Three- and Four-Satellite Continuous Coverage Constellations," *Journal of Guidance, Control, and Dynamics,* Vol. 6, No. 6, 1985, pp. 725–730.

[4]Draim, J. E., "A Common-Period Four-Satellite Continuous Global Coverage Constellation," *Journal of Guidance, Control, and Dynamics,* Vol. 10, No. 5, 1987, pp. 492–499.

[5]Chao, C. C., "An Analytical Integration of the Averaged Equations of Variation Due to Sun-Moon Perturbations and Its Application," The Aerospace Corp., Aerospace TR SD-TR-80-12, El Segundo, CA, Oct. 1979.

[6]Chao, C. C., and Bowen, A. F., "Effects of Long-Term Orbit Perturbations and Injection Errors on GPS Constellation Values," AIAA Paper 86-2173, Aug. 1986.

[7]Chao, C. C., and Baker, J. M., "On the Propagation and Control of Geosynchronous Orbits," *Journal of the Astronautical Sciences,* Vol. 31, No. 1, 1983, pp. 98–115.